Solutions for Selected Exercises

Solutions for selected exercises in the following book are given below.

K. Morita: *Reversible World of Cellular Automata*,
World Scientific Publishing, Singapore (2024).
`https://doi.org/10.1142/13516`

Solution 1.1. Figure 10.1 shows the evolution process of a toad.

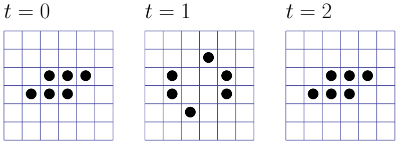

Fig. 10.1 Evolution process of a *toad* in GoL.

Solution 1.2. Let $Q = \{0, 1\}$ be the set of states of the Fredkin's CA. Let α^n denote the configuration at time n (hence α^n is a function $\mathbb{Z}^2 \to Q$). Then, the state transition of the cell at $(x, y) \in \mathbb{Z}^2$ by the local function of Fredkin's CA is expressed by the following formula, where $\oplus$ denotes the mod 2 addition.

$$\alpha^{n+1}(x, y) \;=\; \alpha^n(x, y+1) \oplus \alpha^n(x+1, y) \oplus \alpha^n(x, y-1) \oplus \alpha^n(x-1, y) \quad \text{(i)}$$

Applying (i) repeatedly, we have the equation (ii), since $z \oplus z = 0$ holds for any $z \in \{0,1\}$.

$$\begin{aligned}
&\alpha^{n+2}(x,y)\\
=\ &\alpha^n(x,y+2) \oplus \alpha^n(x+1,y+1) \oplus \alpha^n(x,y) \oplus \alpha^n(x-1,y+1)\\
&\oplus \alpha^n(x+1,y+1) \oplus \alpha^n(x+2,y) \oplus \alpha^n(x+1,y-1) \oplus \alpha^n(x,y)\\
&\oplus \alpha^n(x,y) \oplus \alpha^n(x+1,y-1) \oplus \alpha^n(x,y-2) \oplus \alpha^n(x-1,y-1)\\
&\oplus \alpha^n(x-1,y+1) \oplus \alpha^n(x,y) \oplus \alpha^n(x-1,y-1) \oplus \alpha^n(x-2,y)\\
=\ &\alpha^n(x,y+2) \oplus \alpha^n(x+2,y) \oplus \alpha^n(x,y-2) \oplus \alpha^n(x-2,y) \qquad \text{(ii)}
\end{aligned}$$

Applying (ii) repeatedly, and so on, we obtain the formula (iii) for each $m \in \{1,2,\cdots\}$.

$$\alpha^{n+2^m}(x,y) = \alpha^n(x,y+2^m) \oplus \alpha^n(x+2^m,y) \oplus \alpha^n(x,y-2^m) \oplus \alpha^n(x-2^m,y) \quad \text{(iii)}$$

By (iii), we can see that if 2^m is larger than the diameter (*i.e.*, maximum of the height and the width) of the initial pattern, then four replicated pattern appear at time 2^m.

It is also easy to see that at time

$$n = 2^{m_1} + 2^{m_2} \cdots + 2^{m_k}$$

such that $m_1 > m_2 > \cdots > m_k$ and 2^{m_k} is larger than the diameter of the initial pattern, 4^k copies of the pattern appear.

Solution 1.3. (omitted)

Solution 1.4. (omitted)

Solution 2.1. (1) Figure 10.2 shows its evolution process. Two copies of a space-moving pattern of period 3 appear, and they move opposite directions. Hence, it is diameter-growing.

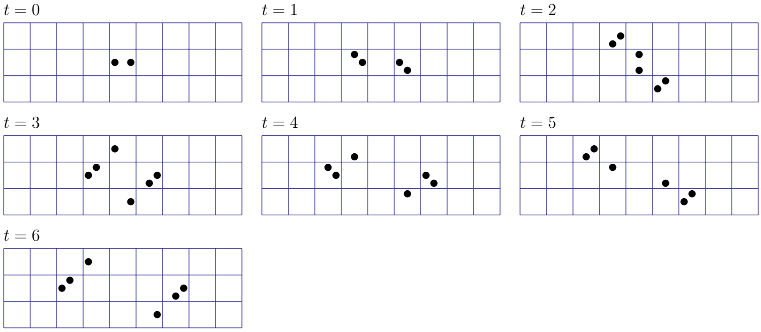

Fig. 10.2 Evolution process of a 2-dot configuration in ESPCA-0945df.

(2) Figure 10.3 shows its evolution process. A space-moving pattern of period 3, and a periodic pattern of period 22 appear. Since the former moves rightward, it is diameter-growing as a whole.

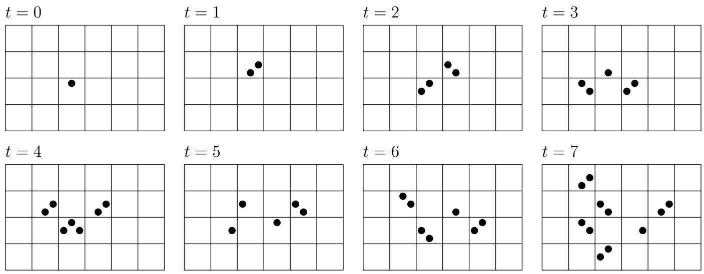

Fig. 10.3 Evolution process of a 1-dot configuration in ESPCA-0945df.

Solution 2.2. (1) Figure 10.4 shows its evolution process. It is a periodic configuration of period 8.

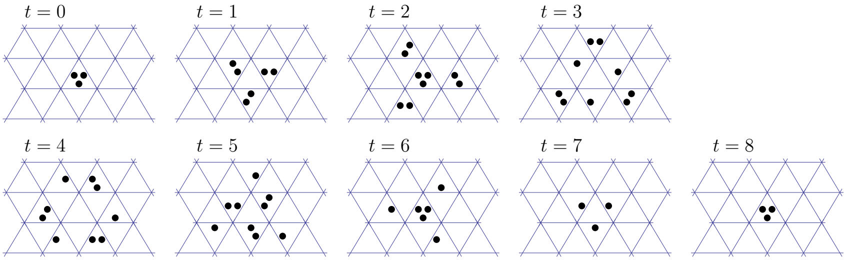

Fig. 10.4 Evolution process of a 3-dot configuration in ETPCA-0347.

(2) The configuration rotates clockwise by 60° in 7 steps (Fig. 10.5). Hence, it is a periodic configuration of period 42.

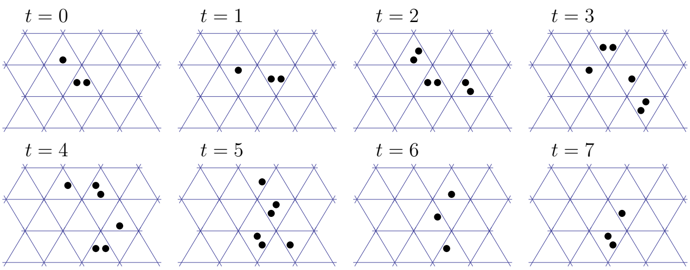

Fig. 10.5 Evolution process of a rotating configuration in ETPCA-0347.

Solution 2.3. *Proof.* Assume, on the contrary, negation of the statement holds:

$$\exists d > 0\ \forall t > 0\ (\mathrm{diam}((F^{-1})^t(\alpha)) \leq d)$$

It means that there are infinitely many instances of $t > 0$ such that $\mathrm{diam}((F^{-1})^t(\alpha)) \leq d$. We can see that the total number of different configurations whose diameter is bounded by d is finite except their translations. Therefore, there exist $0 < t_1 < t_2$ such that $(F^{-1})^{t_1}(\alpha) = (F^{-1})^{t_2}(\alpha)$, or $(F^{-1})^{t_1}(\alpha)$ is a translation of $(F^{-1})^{t_2}(\alpha)$. In the former case $(F^{-1})^{t_2}(\alpha)$ is periodic, while in the latter case it is space-moving. Hence, $\alpha = (F^{-1})^0(\alpha)$ is also periodic or space-moving, and thus not diameter-growing, a contradiction. Therefore, the statement holds.

Solution 2.4. Figure 2.30 (a) is a periodic configuration of period 34. Figure 2.30 (b) is a diameter-growing one with a chaotic core that generates an unbounded number of space-moving patterns of period 3. The above solution is found in the following pattern file:

`Cx_Exercise_2_4_ESPCA-0945df.rle`

Solution 2.5. Figure 2.31 (a) is a periodic configuration of period 24. Figure 2.31 (b) generates two space-moving patterns, which move to the east and the south-east directions. Hence, it is a diameter-growing one. The above solution is found in the following pattern file:

`Cx_Exercise_2_5_ETPCA_0347.rle`

Solution 3.1.

α :

$F_{0347}(\alpha)$:

$\hat{H} \circ F_{0347}(\alpha)$:

$F_{0347} \circ \hat{H} \circ F_{0347}(\alpha)$:

$\hat{H} \circ F_{0347} \circ \hat{H} \circ F_{0347}(\alpha)$:

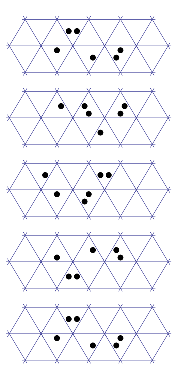

Fig. 10.6 T-symmetry under $\hat{H}$ in reversible ETPCA-0347.

Solution 3.2. (1) Let $\hat{f}$ denote the local function of ESPCA-073a2f. Then,

$$\begin{array}{ll} \hat{f}(0,0,0,0) = (0,0,0,0), & \hat{f}(0,0,1,0) = (0,1,1,1), \\ \hat{f}(0,0,1,1) = (0,0,1,1), & \hat{f}(1,0,1,0) = (1,0,1,0), \\ \hat{f}(0,1,1,1) = (0,0,1,0), & \hat{f}(1,1,1,1) = (1,1,1,1). \end{array}$$

Therefore,

$$\begin{array}{ll} \hat{f}^{-1}(0,0,0,0) = (0,0,0,0), & \hat{f}^{-1}(0,1,1,1) = (0,0,1,0), \\ \hat{f}^{-1}(0,0,1,1) = (0,0,1,1), & \hat{f}^{-1}(1,0,1,0) = (1,0,1,0), \\ \hat{f}^{-1}(0,0,1,0) = (0,1,1,1), & \hat{f}^{-1}(1,1,1,1) = (1,1,1,1). \end{array}$$

Hence, $\hat{f} = \hat{f}^{-1}$. Thus, by Theorem 3.1, ESPCA-073a2f is T-symmetric under H^{rev}.

(2) A solution is found in the following two pattern files:

```
Cx_Exercise_3_2_ESPCA-073a2f_T-symmetry.rle
Cx_Exercise_3_2_ESPCA-08cadf_Hrev.rle
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Solution 3.3. A solution is found in the following pattern file:

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Cx_Exercise_3_2_ESPCA-073a2f_T-symmetry.rle
```

Solution 4.1. (1) $T_{\mathrm{add}} = (Q_{\mathrm{add}}, \{0,1\}, q_0, \{q_f\}, \delta_{\mathrm{add}})$, where

$$\begin{array}{rl} Q_{\mathrm{add}} = & \{\ q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_f\ \}, \\ \delta_{\mathrm{add}} = & \{\ [q_0,0,0,R,q_1],\ [q_1,0,0,L,q_7],\ [q_1,1,0,R,q_2],\ [q_2,0,0,R,q_3], \\ & [q_2,1,1,R,q_2],\ [q_3,0,1,R,q_4],\ [q_3,1,1,R,q_3],\ [q_4,0,0,L,q_5], \\ & [q_5,0,0,L,q_6],\ [q_5,1,1,L,q_5],\ [q_6,0,1,R,q_1],\ [q_6,1,1,L,q_6], \\ & [q_7,0,0,N,q_f],\ [q_7,1,1,L,q_7]\ \ \}. \end{array}$$

See also the solution of Exercise 4.8 given as a pattern file for Golly.

(2) $T_{\mathrm{mult}} = (Q_{\mathrm{mult}}, \{0,1\}, p_0, \{p_f\}, \delta_{\mathrm{mult}})$, where

$$\begin{array}{rl} Q_{\mathrm{mult}} = & \{\ p_0, p_1, p_2, p_3, p_4, p_f, q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\ \}, \\ \delta_{\mathrm{mult}} = & \{\ [p_0,0,0,R,p_1],\ [p_1,0,0,L,p_4],\ [p_1,1,0,R,p_2],\ [p_2,0,0,R,q_1], \\ & [p_2,1,1,R,p_2],\ [p_3,0,1,R,p_1],\ [p_3,1,1,L,p_3],\ [p_4,0,0,N,p_f], \\ & [p_4,1,1,L,p_4],\ [q_1,0,0,L,q_7],\ [q_1,1,0,R,q_2],\ [q_2,0,0,R,q_3], \\ & [q_2,1,1,R,q_2],\ [q_3,0,1,R,q_4],\ [q_3,1,1,R,q_3],\ [q_4,0,0,L,q_5], \\ & [q_5,0,0,L,q_6],\ [q_5,1,1,L,q_5],\ [q_6,0,1,R,q_1],\ [q_6,1,1,L,q_6], \\ & [q_7,0,0,L,p_3],\ [q_7,1,1,L,q_7]\ \ \}. \end{array}$$

See also the solution of Exercise 4.8 given as a pattern file for Golly.

Solution 4.2. (1) $M_{\text{half}} = (Q_{\text{half}}, 2, \delta_{\text{half}}, h_0, \{h_f\})$, where

$$\begin{aligned}
Q_{\text{half}} &= \{\ h_0, h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_f\ \},\\
\delta_{\text{half}} &= \{\ [h_0, 2, Z, h_1],\ [h_1, 1, Z, h_f],\ [h_1, 1, P, h_2],\ [h_2, 1, -, h_3],\\
&\qquad [h_3, 1, Z, h_4],\ [h_3, 1, P, h_6],\ [h_4, 1, +, h_5],\ [h_5, 1, P, h_f],\\
&\qquad [h_6, 1, -, h_7],\ [h_7, 2, +, h_8],\ [h_8, 2, P, h_1]\ \ \}.
\end{aligned}$$

See also the solution of Exercise 4.9 given as a pattern file for Golly.

(2) P_{half} is shown in Fig. 10.7.

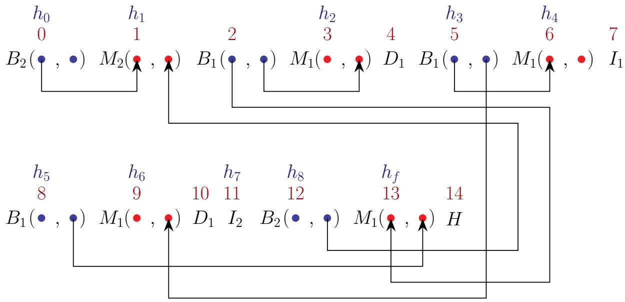

Fig. 10.7 WFP P_{half}.

See also the solution of Exercise 4.9 given as a pattern file for Golly.

Solution 4.3. See the solution of Exercise 4.10 given as a pattern file for Golly.

Solution 4.4. See the solution of Exercise 4.10 given as a pattern file for Golly.

Solution 4.5. See the solution of Exercise 4.10 given as a pattern file for Golly.

Solution 4.6. See the solution of Exercise 4.10 given as a pattern file for Golly.

Solution 4.7. See the solution of Exercise 4.10 given as a pattern file for Golly.

Solution 4.8. A solution is found in the following pattern file:
`Cx_Exercise_4_08_RTMs.rle`

Solution 4.9. A solution is found in the following pattern file:
`Cx_Exercise_4_09_RCMs.rle`

Solution 4.10. A solution is found in the following pattern file:

`Cx_Exercise_4_10_RLEMs.rle`

Solution 5.1. Figure 10.8 shows the evolution process.

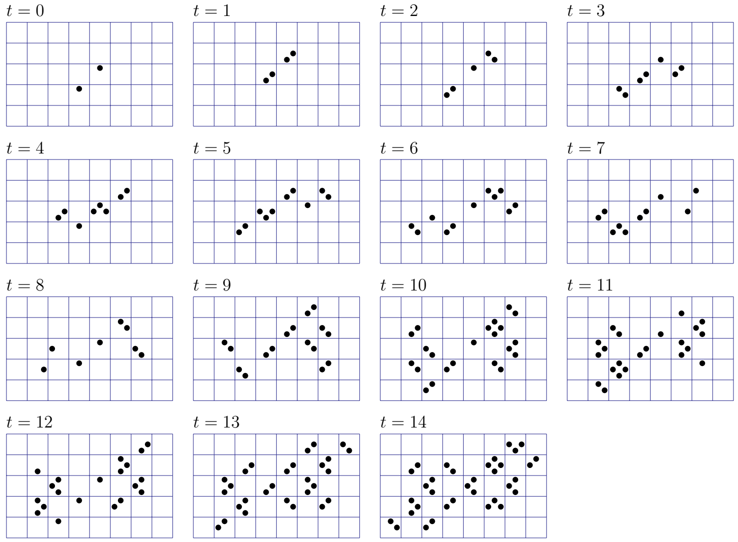

Fig. 10.8 Evolution process of the periodic pattern of period 60 in ESPCA-0945df.

Solution 5.2. First note that ESPCA-05f050 is defined by $(a_1, a_2, a_3, a_4) = (0, 1, 0, 1)$ (see Table 5.1). Let f be the local function of ESPCA-05f050. If $f(t, r, b, l) = (t', r', b', l')$ for $(t, r, b, l), (t', r', b', l') \in \{0, 1\}^4$, then the following holds, where $\oplus$ denotes the mod 2 addition.

$$t' = r \oplus l, \quad r' = b \oplus t, \quad b' = l \oplus r, \quad l' = t \oplus b$$

Let α^n denote the configuration at time n (it is a function $\mathbb{Z}^2 \to \{0, 1\}^4$). Thus, $\alpha^n(x, y)$ is the state of the cell at $(x, y) \in \mathbb{Z}^2$. Let $\alpha^n_{\mathrm{t}}(x, y)$, $\alpha^n_{\mathrm{r}}(x, y)$, $\alpha^n_{\mathrm{b}}(x, y)$, and $\alpha^n_{\mathrm{l}}(x, y)$ denote the states of the top, right, bottom, and left parts of the cell at (x, y), respectively. Then the following holds.

$$\begin{aligned}
\alpha^{n+1}_{\mathrm{t}}(x, y) &= \alpha^n_{\mathrm{r}}(x-1, y) \oplus \alpha^n_{\mathrm{l}}(x+1, y) \\
\alpha^{n+1}_{\mathrm{r}}(x, y) &= \alpha^n_{\mathrm{b}}(x, y+1) \oplus \alpha^n_{\mathrm{t}}(x, y-1) \\
\alpha^{n+1}_{\mathrm{b}}(x, y) &= \alpha^n_{\mathrm{l}}(x+1, y) \oplus \alpha^n_{\mathrm{r}}(x-1, y) \\
\alpha^{n+1}_{\mathrm{l}}(x, y) &= \alpha^n_{\mathrm{t}}(x, y-1) \oplus \alpha^n_{\mathrm{b}}(x, y+1)
\end{aligned}$$

Using the above equations, we have the following.

$$\begin{aligned}
\alpha_{\rm t}^{n+2}(x,y) &= \alpha_{\rm r}^{n+1}(x-1,y)\oplus\alpha_{\rm l}^{n+1}(x+1,y)\\
&= \alpha_{\rm b}^{n}(x-1,y+1)\oplus\alpha_{\rm t}^{n}(x-1,y-1)\oplus\alpha_{\rm t}^{n}(x+1,y-1)\oplus\alpha_{\rm b}^{n}(x+1,y+1)\\
\alpha_{\rm r}^{n+2}(x,y) &= \alpha_{\rm b}^{n+1}(x,y+1)\oplus\alpha_{\rm t}^{n+1}(x,y-1)\\
&= \alpha_{\rm l}^{n}(x+1,y+1)\oplus\alpha_{\rm r}^{n}(x-1,y+1)\oplus\alpha_{\rm r}^{n}(x-1,y-1)\oplus\alpha_{\rm l}^{n}(x+1,y-1)\\
\alpha_{\rm b}^{n+2}(x,y) &= \alpha_{\rm l}^{n+1}(x+1,y)\oplus\alpha_{\rm r}^{n+1}(x-1,y)\\
&= \alpha_{\rm t}^{n}(x+1,y-1)\oplus\alpha_{\rm b}^{n}(x+1,y+1)\oplus\alpha_{\rm b}^{n}(x-1,y+1)\oplus\alpha_{\rm t}^{n}(x-1,y-1)\\
\alpha_{\rm l}^{n+2}(x,y) &= \alpha_{\rm t}^{n+1}(x,y-1)\oplus\alpha_{\rm b}^{n+1}(x,y+1)\\
&= \alpha_{\rm r}^{n}(x-1,y-1)\oplus\alpha_{\rm l}^{n}(x+1,y-1)\oplus\alpha_{\rm l}^{n}(x+1,y+1)\oplus\alpha_{\rm r}^{n}(x-1,y+1)
\end{aligned}$$

Again, using the above equations, we have the following.

$$\begin{aligned}
\alpha_{\rm t}^{n+4}(x,y) &= \alpha_{\rm b}^{n+2}(x-1,y+1)\oplus\alpha_{\rm t}^{n+2}(x-1,y-1)\oplus\\
&\quad\ \alpha_{\rm t}^{n+2}(x+1,y-1)\oplus\alpha_{\rm b}^{n+2}(x+1,y+1)\\
&= \alpha_{\rm t}^{n}(x,y)\oplus\alpha_{\rm b}^{n}(x,y+2)\oplus\alpha_{\rm b}^{n}(x-2,y+2)\oplus\alpha_{\rm t}^{n}(x-2,y)\oplus\\
&\quad\ \alpha_{\rm b}^{n}(x-2,y)\oplus\alpha_{\rm t}^{n}(x-2,y-2)\oplus\alpha_{\rm t}^{n}(x,y-2)\oplus\alpha_{\rm b}^{n}(x,y)\oplus\\
&\quad\ \alpha_{\rm b}^{n}(x,y)\oplus\alpha_{\rm t}^{n}(x,y-2)\oplus\alpha_{\rm t}^{n}(x+2,y-2)\oplus\alpha_{\rm b}^{n}(x+2,y)\oplus\\
&\quad\ \alpha_{\rm t}^{n}(x+2,y)\oplus\alpha_{\rm b}^{n}(x+2,y+2)\oplus\alpha_{\rm b}^{n}(x,y+2)\oplus\alpha_{\rm t}^{n}(x,y)\oplus\\
&= \alpha_{\rm b}^{n}(x-2,y+2)\oplus\alpha_{\rm t}^{n}(x-2,y-2)\oplus\alpha_{\rm t}^{n}(x+2,y-2)\oplus\alpha_{\rm b}^{n}(x+2,y+2)
\end{aligned}$$

The above equation is derived from the following fact. From the local function of ESPCA-05f050, we can see that the configurations at time $n > 0$ consist only of the states (0,0,0,0), (0,1,0,1), (1,0,1,0) and (1,1,1,1), as well as the initial configuration. Therefore, in the above formula, for example, $\alpha_{\rm t}^{n}(x-2,y) = \alpha_{\rm b}^{n}(x-2,y)$ holds, and they are cancelled by $\oplus$.

We can derive the following equations in a similar way.

$$\begin{aligned}
\alpha_{\rm r}^{n+4}(x,y) &= \alpha_{\rm l}^{n}(x+2,y+2)\oplus\alpha_{\rm r}^{n}(x-2,y+2)\oplus\alpha_{\rm r}^{n}(x-2,y-2)\oplus\alpha_{\rm l}^{n}(x+2,y-2)\\
\alpha_{\rm b}^{n+4}(x,y) &= \alpha_{\rm t}^{n}(x+2,y-2)\oplus\alpha_{\rm b}^{n}(x+2,y+2)\oplus\alpha_{\rm b}^{n}(x-2,y+2)\oplus\alpha_{\rm t}^{n}(x-2,y-2)\\
\alpha_{\rm l}^{n+4}(x,y) &= \alpha_{\rm r}^{n}(x-2,y-2)\oplus\alpha_{\rm l}^{n}(x+2,y-2)\oplus\alpha_{\rm l}^{n}(x+2,y+2)\oplus\alpha_{\rm r}^{n}(x-2,y+2)
\end{aligned}$$

Repeating this procedure, we have the following for each $m \in \{1,2,\ldots\}$.

$$\begin{aligned}
\alpha_{\rm t}^{n+2^{m+1}}(x,y) &= \alpha_{\rm b}^{n}(x-2^m,y+2^m)\oplus\alpha_{\rm t}^{n}(x-2^m,y-2^m)\oplus\\
&\quad\ \alpha_{\rm t}^{n}(x+2^m,y-2^m)\oplus\alpha_{\rm b}^{n}(x+2^m,y+2^m)\\
\alpha_{\rm r}^{n+2^{m+1}}(x,y) &= \alpha_{\rm l}^{n}(x+2^m,y+2^m)\oplus\alpha_{\rm r}^{n}(x-2^m,y+2^m)\oplus\\
&\quad\ \alpha_{\rm r}^{n}(x-2^m,y-2^m)\oplus\alpha_{\rm l}^{n}(x+2^m,y-2^m)\\
\alpha_{\rm b}^{n+2^{m+1}}(x,y) &= \alpha_{\rm t}^{n}(x+2^m,y-2^m)\oplus\alpha_{\rm b}^{n}(x+2^m,y+2^m)\oplus\\
&\quad\ \alpha_{\rm b}^{n}(x-2^m,y+2^m)\oplus\alpha_{\rm t}^{n}(x-2^m,y-2^m)\\
\alpha_{\rm l}^{n+2^{m+1}}(x,y) &= \alpha_{\rm r}^{n}(x-2^m,y-2^m)\oplus\alpha_{\rm l}^{n}(x+2^m,y-2^m)\oplus\\
&\quad\ \alpha_{\rm l}^{n}(x+2^m,y+2^m)\oplus\alpha_{\rm r}^{n}(x-2^m,y+2^m)
\end{aligned}$$

By above, if 2^m is larger than the size of the initial pattern, then four copies of it appear at time 2^{m+1}. For example, if the initial pattern lies in the area $0 < x < 2^m$ and $0 < y < 2^m$, then $\alpha^0(x-2^m, y+2^m) = \alpha^0(x-2^m, y-2^m) = \alpha^0(x+2^m, y-2^m) = (0,0,0,0)$ for all x and y such that $-2^m < x < 0$ and $-2^m < y < 0$. Thus, the following holds for

$-2^m < x < 0$ and $-2^m < y < 0$.

$$\begin{aligned}
\alpha_{\mathrm{t}}^{2^{m+1}}(x,y) &= \alpha_{\mathrm{b}}^{0}(x+2^m, y+2^m) = \alpha_{\mathrm{t}}^{0}(x+2^m, y+2^m)\\
\alpha_{\mathrm{r}}^{2^{m+1}}(x,y) &= \alpha_{\mathrm{l}}^{0}(x+2^m, y+2^m) = \alpha_{\mathrm{r}}^{0}(x+2^m, y+2^m)\\
\alpha_{\mathrm{b}}^{2^{m+1}}(x,y) &= \alpha_{\mathrm{b}}^{0}(x+2^m, y+2^m)\\
\alpha_{\mathrm{l}}^{2^{m+1}}(x,y) &= \alpha_{\mathrm{l}}^{0}(x+2^m, y+2^m)
\end{aligned}$$

Therefore, a copy of the pattern in the region $0 < x < 2^m$ and $0 < y < 2^m$ at time 0 appears in the region $-2^m < x < 0$ and $-2^m < y < 0$ (*i.e.*, to the south-west direction from the original) at time 2^{m+1}. By a similar argument, copies of the initial pattern also appear to the north-west, north-east and south-east directions.

Solution 5.3. (omitted)

Solution 5.4. A solution is found in the following pattern file:
`Cx_Exercise_5_4_ETPCA-0347_12w_glider_gun.rle`

Solution 6.1. The process of a left-turn is shown in Fig. 10.9.

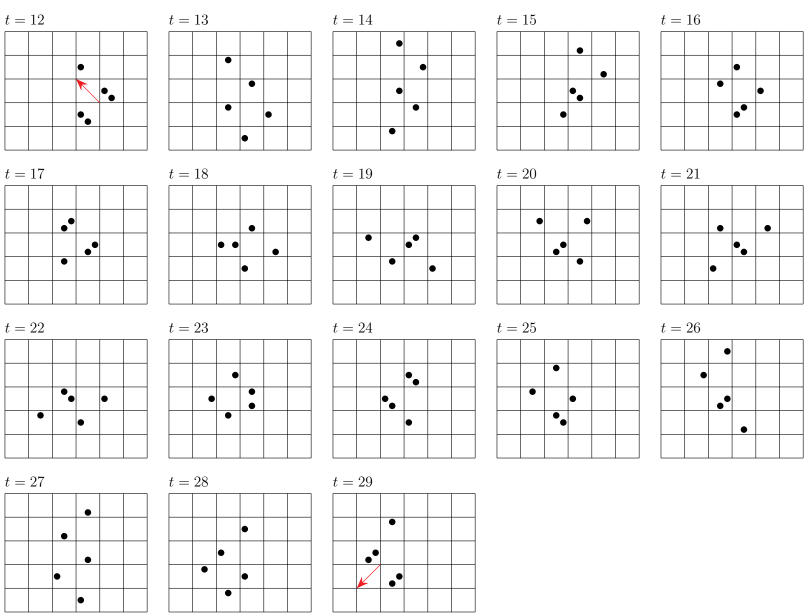

Fig. 10.9 Left-turn process of a glider-12 by a rotor in ESPCA-01c5ef.

Solution 6.2. See the solution of Exercise 6.7 given as a pattern file for Golly.

Solution 6.3. See the solution of Exercise 6.8 given as a pattern file for Golly.

Solution 6.4. A solution is given in Fig. 10.10. Here, every delay element has a unit-time delay. An input should be given at $t \equiv 0 \pmod 2$. See also the solution of Exercise 6.9 given as a pattern file for Golly.

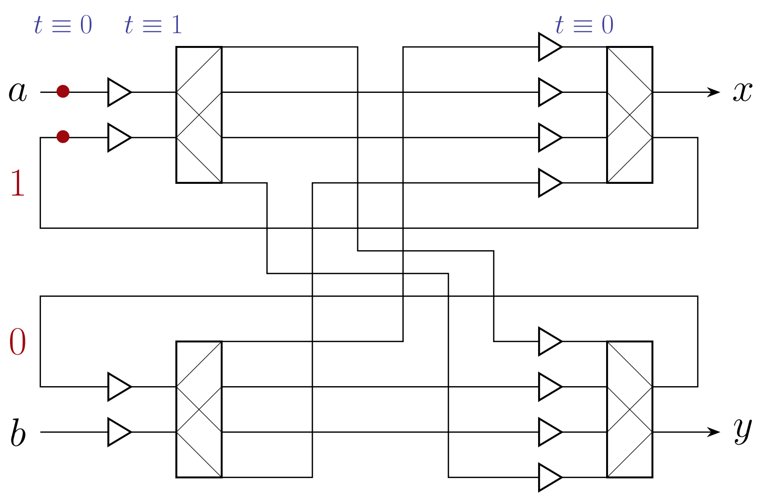

Fig. 10.10 RLEM 2-2 composed of I-gates, I^{-1}-gates, and delay elements. Here, $t \equiv n$ means t and n are congruent modulo 2, and shows that signals can be at the position only when $t \equiv n$.

Solution 6.5. A solution is shown in Fig. 10.11. Here, a number in each delay element shows the delay time. An input should be given at $t \equiv 0$ (mod 6). See also the solution of Exercise 6.9 given as a pattern file for Golly.

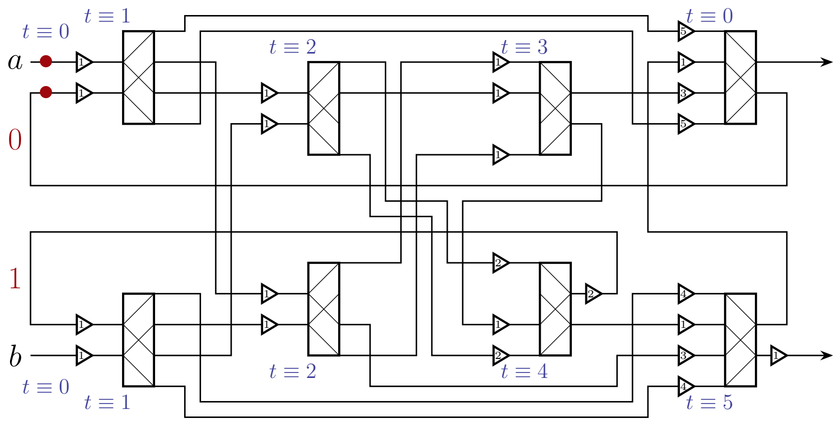

Fig. 10.11 RLEM 2-3 composed of I-gates, I^{-1}-gates, and delay elements. Here, $t \equiv n$ means that t and n are congruent modulo 6.

Another solution: First, note that RLEM 2-3 is realized using only one RE as shown in Fig. 10.12. Therefore, a circuit for RLEM 2-3 is obtained by adding two feedback loops to the circuit of Fig. 6.36.

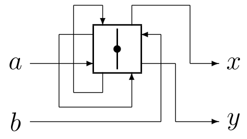

Fig. 10.12 RLEM 2-3 composed of an RE. This figure shows the state 0 of RLEM 2-3.

Solution 6.6. A solution is found in the following pattern file:
`Cx_Exercise_6_6_ESPCA-01caef_RTM_square_by_RE.mc`

Solution 6.7. A solution is found in the following pattern file:
`Cx_Exercise_6_7_ESPCA-01c5ef_RLEM_2-2.rle`

Solution 6.8. A solution is found in the following pattern file:
`Cx_Exercise_6_8_ESPCA-01caef_RLEM_2-3.rle`

Solution 6.9. A solution is found in the following pattern file:
`Cx_Exercise_6_9_ESPCA-02c5df_RLEM_2-2_2-3.rle`

Solution 7.1. As shown in Fig. 10.13, the fin rotates around the block by 120° in 14 steps. Hence, it is a periodic pattern of period 42.

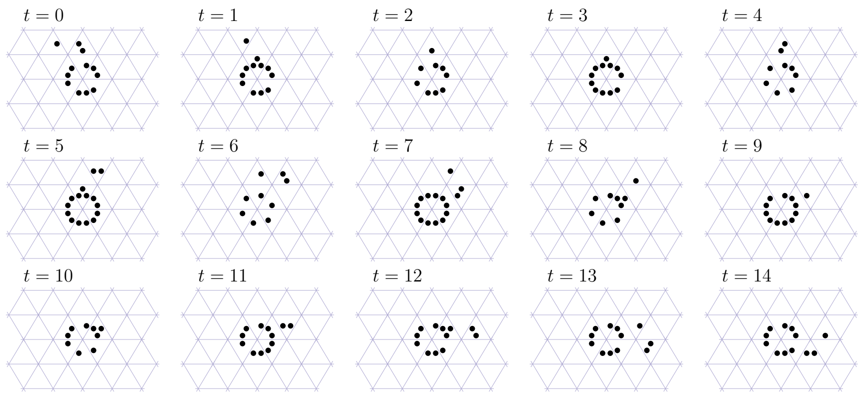

Fig. 10.13 Interaction of a fin and a block in ETPCA-0347.

Solution 7.2. The evolving process is given in Fig. 10.14.

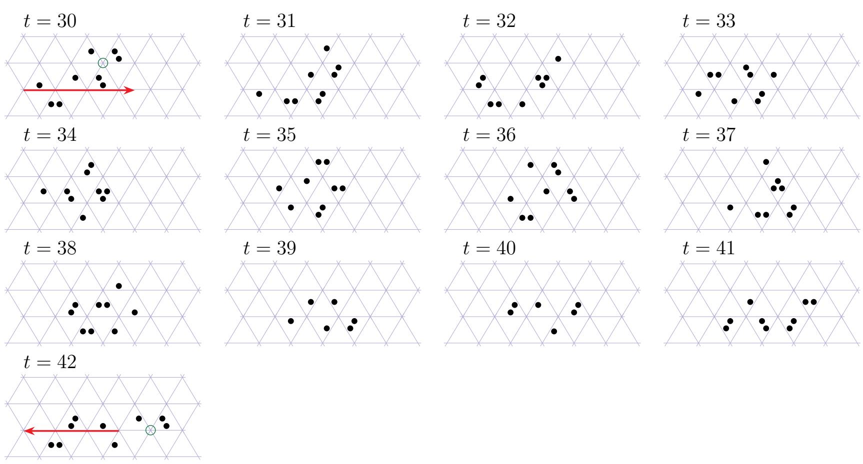

Fig. 10.14 Pushing process of a fin by a glider in ETPCA-0347.

Solution 7.3. A solution is shown in Fig. 10.15. See also the solution of Exercise 7.5 given as a pattern file for Golly.

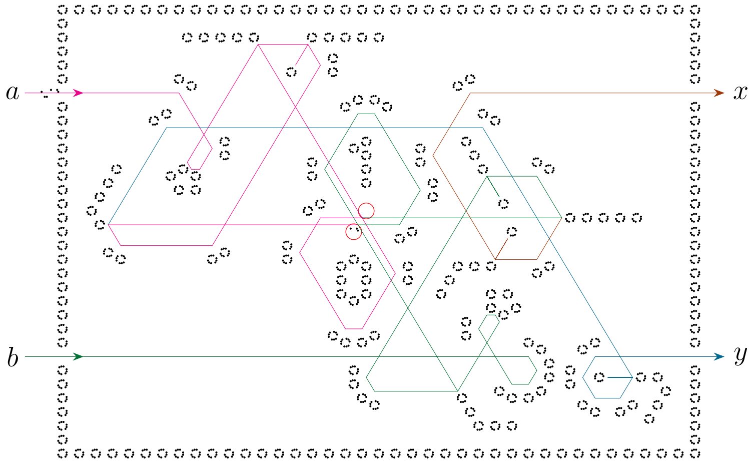

Fig. 10.15 RLEM 2-2 implemented in ETPCA-0347.

Solution 7.4. (omitted)

Solution 7.5. A solution is found in the following pattern file:

`Cx_Exercise_7_5_ETPCA-0347_RLEM_2-2.rle`

Solution 8.1. See the solution of Exercise 8.5 given as a pattern file for Golly.

Solution 8.2. (1) Figure 10.16 shows that it is a space-moving pattern.

(2) $c/7$

(3) See the solution of Exercise 8.6 given as a pattern file for Golly.

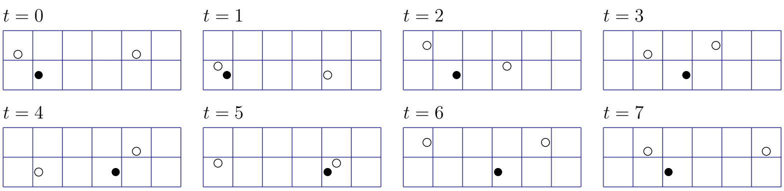

Fig. 10.16 Evolving process of a space-moving pattern in the SPCA P_3.

Solution 8.3.

(1) $3^3 \times 9^3 \times 81^{18} = 3 \times 81^{20}$
$= 443426488243037769948249630619149892803$

(2) $3! \times 6 \times 4 \times 2 \times 18! \times 4^{18}$
$= 126710718026674580911816704000$

Solution 8.4. See the solution of Exercise 8.7 given as a pattern file for Golly.

Solution 8.5. A solution is found in the following pattern file:
`Cx_Exercise_8_5_P3_B-turn.rle`

Solution 8.6. A solution is found in the following pattern file:
`Cx_Exercise_8_6_P3_Slow_space_moving.rle`

Solution 8.7. A solution is found in the following pattern file:
`Cx_Exercise_8_7_P3_RCM_half.rle`

Solution 9.1. Construct the following RCM(3) M_{loop} in P_3 using the RCM(3) M_{exp} given in Example 8.2. If $x \in \mathbb{N}$ is given in the first counter, M_{loop} first simulates M_{exp}, and writes 2^x in the second counter. Next, it retraces the computing process backward. By this, the second counter is cleared, and the contents of the first counter becomes x. Then, repeats this procedure indefinitely. Therefore, M_{loop} is a periodic pattern in P_3. The pattern file `Cx_Exercise_9_1_P3_RCM_loop.rle` for Golly contained in [7] shows M_{loop} in P_3. From this pattern, we can see that if M_{loop} keeps a triplet (x_1, x_2, x_3) in its three counters, then the diameter of the pattern is $615 + 2 \cdot \max\{x_1, x_2, x_3\}$. Therefore, if $x \in \mathbb{N}$ is given to M_{loop}, $d_{\min} = 615 + 2x$, and $d_{\max} = 615 + 2 \cdot 2^x$. Therefore, $d_{\max}/d_{\min} = (615 + 2 \cdot 2^x)/(615 + 2x)$, and thus $d_{\max}/d_{\min} > r$ holds for sufficiently large x.

Solution 9.2 (omitted)